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OVERVIEW OF NEUTRINO MIXING MODELS AND WAYS TO DIFFERENTIATE AMONG THEM

CARL H. ALBRIGHT

Department of Physics, Northern Illinois University, DeKalb, IL 60115, USA

and

Theoretical Physics Department, Fermi National Accelerator Laboratory, Batavia, IL 60510, USA

E-mail: albright@fnal.gov

ABSTRACT

An overview of neutrino-mixing models is presented with emphasis on the types of horizontal flavor and vertical family symmetries that have been invoked. Distributions for the mixing angles of many models are displayed. Ways to differentiate among the models and to narrow the list of viable models are discussed.

1. Introduction

Several hundred models of neutrino masses and mixings can be found in the literature which purport to explain the known oscillation data and predict the currently unknown quantities. We present an overview of the types of models proposed and discuss ways in which the list of viable models can be reduced when more precise data is obtained. This presentation is an update of one published in 2006 in collaboration with Mu-Chun Chen[1)].

2. Present Oscillation Data and Unknowns

The present data within 3σ accuracy as determined by Fogli et al.[2)], for example, is given be

$$\begin{aligned}
\Delta m^2_{32} &= 2.39 \begin{array}{l} +0.42 \\ -0.33 \end{array} \times 10^{-3}\ \mathrm{eV}^2, \\
\Delta m^2_{21} &= 7.67 \begin{array}{l} +0.52 \\ -0.53 \end{array} \times 10^{-5}\ \mathrm{eV}^2, \\
\sin^2\theta_{23} &= 0.466 \begin{array}{l} +0.178 \\ -0.135 \end{array}, \\
\sin^2\theta_{12} &= 0.312 \begin{array}{l} +0.063 \\ -0.049 \end{array}, \\
\sin^2\theta_{13} &\leq 0.046, \quad (0.016 \pm 0.010),
\end{aligned} \tag{1}$$

where the last figure in parenthesis indicates a departure of the reactor neutrino angle from zero with one σ accuracy determination. The data suggests the approximate tri-bimaximal mixing texture of Harrison, Perkins, and Scott[3],

$$U_{PMNS} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}, \tag{2}$$

with $\sin^2\theta_{23} = 0.5,\ \sin^2\theta_{12} = 0.33$, and $\sin^2\theta_{13} = 0$.

The reason for the plethera of models still in agreement with experiment of course can be traced to the inaccuracy of the present data and the imprecision of the model predictions in many cases. In addition, there are a number of unknowns that must still be determined: the hierarchy and absolute mass scales of the light neutrinos; the Dirac or Majorana nature of the neutrinos; the CP-violating phases of the mixing matrix; how close to zero the reactor neutrino angle, θ_{13}, lies; how near maximal the atmospheric neutrino mixing angle is; whether the approximate tri-bimaximal mixing is a softly-broken or an accidental symmmetry; whether neutrino-less double beta decay will be observable, and how large charged lepton flavor violation will turn out to be. In this presentation we survey the models to determine what they predict for the mixing angles, hierarchy, and briefly mention the role charged lepton flavor violation can play.

3. Theoretical Framework

The observation of neutrino oscillations implies that neutrinos have mass, with the mass squared differences given in Eq.(1). Information concerning the absolute neutrino mass scale has been determined by the combined WMAP, SDSS, and Lyman alpha data which place an upper limit on the sum of the masses[4],

$$\sum_i m_i \leq 0.17 - 1.2 \text{ eV}, \tag{3}$$

depending upon the conservative nature of the bound extracted. An extension of the SM is then required, and possible approaches include one or more of the following:

- the introduction of dim-5 effective non-renormalizable operators;
- the addition of right-handed neutrinos with their Yukawa couplings to the left-handed neutrinos;
- the addition of direct mass terms with right-handed Majorana couplings;
- the addition of a Higgs triplet with left-handed Majorana couplings;
- the addition of a fermion triplet with Higgs doublet couplings.

If we exclude the last possibility, the general 6×6 neutrino mass matrix in the $B(\nu_{\alpha L},\ N^c_{\alpha L})$ flavor basis of the six left-handed fields then has the following structure in terms of 3×3 submatrices:

$$\mathcal{M} = \begin{pmatrix} M_L & M_N^T \\ M_N & M_R \end{pmatrix}, \tag{4}$$

where M_N is the Dirac neutrino mass matrix, M_L the left-handed and M_R the right-handed Majorana neutrino mass matrices. With $M_L = 0$ and $M_N << M_R$ the type I seesaw formula,

$$m_\nu = -M_N^T M_R^{-1} M_N, \tag{5}$$

is obtained for the light left-handed Majorana neutrinos, while if $M_L \neq 0$ and $M_N << M_R$, one obtains the type II seesaw formula,

$$m_\nu = M_L - M_N^T M_R^{-1} M_N. \tag{6}$$

There are two main approaches which we now describe that one can pursue to learn more about the theory behind the lepton mass generation.

3.1. Top - Down Approach

In the top-down approach one postulates the form of the mass matrix from first principles. The models will differ then due to the horizontal flavor symmetry chosen, the vertical family symmetry (if any) selected, and the fermion and Higgs representation assignments made.

The effective light left-handed Majorana mass matrix m_ν is constructed directly or with the seesaw formula once the Dirac neutrino matrix M_N and the Majorana neutrino matrices M_R (and M_L) are specified. Since m_ν is complex symmetric, it can be diagonalized by a unitary transformation, U_{ν_L}, to give

$$m_\nu^{diag} = U_{\nu_L}^T m_\nu U_{\nu_L} = \mathrm{diag}(m_1,\ m_2,\ m_3), \tag{7}$$

with real, positive masses down the diagonal. On the other hand, the Dirac charged lepton mass matrix is diagonalized by a bi-unitary transformation according to

$$m_\ell^{diag} = U_{\ell R}^\dagger m_\ell U_{\ell L} = \mathrm{diag}(m_e,\ m_\mu, m_\tau). \tag{8}$$

The neutrino mixing matrix[5)], V_{PMNS}, is then given by

$$V_{PMNS} \equiv U_{\ell L}^\dagger U_{\nu_L} = U_{PMNS}\Phi, \tag{9}$$

in the lepton flavor basis with $\Phi = \text{diag}(1, e^{i\alpha},\ e^{i\beta})$. Note that the Majorana phase matrix Φ is required in order to compensate for any phase rotation on U_{ν_L} needed to bring it into the Particle Data Book phase convention[6].

3.2. Bottom - Up Approach

On the other hand, with a bottom-up approach in the diagonal lepton flavor basis and with the general PMNS mixing matrix, one can determine the general texture of the light neutrino mass matrix to be

$$
\begin{aligned}
M_\nu &= U^*_{PMNS}\Phi^* M_\nu^{\text{diag}}\Phi^* U^\dagger_{PMNS} \\
&= U^*_{PMNS}\text{diag}(m_1,\ m_2 e^{-2i\alpha},\ m_3 e^{-2i\beta})U^\dagger_{PMNS} \\
&\equiv \begin{pmatrix} A & B & B' \\ \cdot & F' & E \\ \cdot & \cdot & F \end{pmatrix},
\end{aligned} \tag{10}
$$

where the matrix elements are expressed in terms of the unknown neutrino masses, mixing angles and phases. By restricting the mixing matrix, one can learn that some of the matrix elements may not be independent.

4. Models and Mixing Angle Predictions

When the first hints of atmospheric neutrino oscillations were discovered around 1992 by the Super-Kamiokande Collaboration[7], it became fashionable to assign texture zeros in different positions to m_ν with a top-down approach in hopes of identifying some flavor symmetry, but the procedure is basis dependent[8].

Another popular method invoked a $L_e - L_\mu - L_\tau$ lepton flavor symmetry[9]. The mass matrix then assumes the following form

$$
m_\nu = \begin{pmatrix} 0 & * & * \\ \cdot & 0 & 0 \\ \cdot & 0 & 0 \end{pmatrix}, \tag{11}
$$

which only leads to an inverted hierarchy.

By making use of a bottom-up approach instead, one is able to observe that a $\mu\ -\ \tau$ interchange symmetry with $B' = B,\ F' = F$ in Eq. (10) leads to $\sin^2\theta_{23} = 0.5,\ \sin^2\theta_{13} = 0$ with $\sin^2\theta_{12}$ arbitrary.

On the other hand, with the assumption of exact tri-bimaximal mixing for which $\sin^2\theta_{23} = 0.5,\ \sin^2\theta_{13} = 0$, and $\sin^2\theta_{12} = 0.333$, one finds in Eq. (10) that $B' = B,\ F' = F = \frac{1}{2}(A + B + D)$ and $E = \frac{1}{2}(A + B - D)$, so that just three unknowns are present.

With the realization in the past five years that neutrino mixing is well approximated by the tri-bimaximal mixing matrix, the name of the game has become one of finding what discrete horizontal flavor symmetry groups would lead naturally to this mixing pattern. Such flavor symmetries can then be used as starting points with soft breaking as the next approximation.

4.1. Discrete Horizontal Flavor Symmetry Groups

Of special interest are those groups containing doublet and triplet irreducible representations. We list several of the well-studied groups and pertinent features of each.

The permutation group of three objects, S_3, contains 6 elements with $1, 1'$, and 2 dimensional irreducible representations (IR's). The same eigenstates occur as those for tri-bimaximal mixing, but there is a 2-fold neutrino mass degeneracy.

The group A_4 of even permutations of four objects has 12 elements with IR's labeled $1, 1', 1''$, and 3. A $U(1)_{FN}$ flavon group[10] is often imposed to fix the mass scale which is otherwise scale-independent. Early attempts to extend this flavor group to the quark sector failed, as the CKM mixing matrix for the quarks remained diagonal.

The group T' is the covering group of A_4, but interestingly A_4 is not one of its subgroups. It contains 24 elements with $1, 1', 1'', 3, 2, 2', 2''$ IR's, where the first four are identical to those in A_4. While tri-bimaximal mixing is obtained for the leptons, due to the presence of the three doublet IR's, a satisfactory CKM mixing matrix can also be obtained for the quarks.

The permutation group of 4 objects, S_4, has 24 elements with $1, 1', 2, 3, 3'$ IR's. Lam has proved this is the smallest symmetry group naturally related to tri-bimaximal mixing, if one requires all IR's to participate in the model[11].

4.2. Examples Involving GUT Models

Studies of neutrino mixing models in the framework of grand unified theories with a vertical family symmetry were first pursued in the 1990's and more intensely following the discovery of atmospheric neutrino oscillations by the Super-Kamiokande Collaboration in 1998. Examples exist of models based on $SU(5)$, $SO(10)$, and E_6, where the $SO(10)$ models are generally of two types.

The so-called "minimal" $SO(10)$ models[12] involve Higgs fields appearing in the **10** and **126** IR's, but newer models of this type have been extended to include the **120**, **45**, and/or **54** IR's. They generally result in symmetric and/or antisymmetric contributions to the quark and lepton mass matrices.

On the other hand, $SO(10)$ models[13] with Higgs fields in the $\mathbf{10}, \mathbf{16}, \overline{\mathbf{16}}$ and **45** IR's result in "lopsided" down quark and charged lepton mass matrices due to the $SU(5)$ structure of the electroweak VEV's appearing in the **16** and $\overline{\mathbf{16}}$ representations.

For either type of GUT model, type I seesaws only lead to a stable normal hierarchy for the light neutrino masses[14], while type I + II seesaws can also result in an inverted hierarchy. Most of the $SO(10)$ models have a continuous and/or discrete flavor symmetry group producted with them, but no efforts were initially made to introduce a discrete flavor symmetry group of the type discussed earlier. A few examples can now be found in the literature which combine an $SU(5)$, $SO(10)$ or E_6 GUT symmetry with a T' or A_4 flavor symmetry with some success[15].

Table 1: Mixing Angles for Models with Lepton Flavor Symmetry.

Reference	Hierarchy	$\sin^2 2\theta_{23}$	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$
Texture Zero Models:					
GL1 [16]	NH	1.0		≥ 0.005	
WY [17]	NH			0.0006 - 0.0030	
	IH			0.0006 - 0.0030	
	NH			< 0.023	
	NH			0.017 - 0.14	
CPP [18]	NH			0.0066 - 0.0083	
	IH			≥ 0.00005	
	IH			≥ 0.032	
$\mathbf{L_e - L_\mu - L_\tau}$ **Models:**					
BM [19]	IH			0.00029	
GMN1 [20]	IH		≥ 0.28	≤ 0.05	
PR [21]	IH		$\lesssim 0.37$	≥ 0.007	
GL2 [22]	IH		0.30	0	
2-3 Symmetric Models:					
RS [23]	NH	$\theta_{23} \leq 45°$		0	
	IH	$\theta_{23} \geq 45°$		≤ 0.02	
MN [24]	NH	1.0		0.0024	
AKKL [25]	NH			0.006 - 0.016	
	IH			0.022 - 0.04	
SRB [26]	IH	1.0	0.31	0	0.50
BY [27]	NH	1.0	0.33	< 0.0025	
	IH	1.0	0.33	< 0.008	
$\mathbf{S_3}$ **Models:**					
KMMR-J [28]	IH	1.0		0.000012	
CFM [29]	NH			0.00006 - 0.001	
T [30]	NH			0.0016 - 0.0036	
TY [31]	IH	0.93	0.30	0.0025	0.37
MNY [32]	NH			0.000004 - 0.000036	
MMP [33]	IH	1.0	0.31	0.0034	
MC [34]	NH	1.0		< 0.01	

Reference	Hierarchy	$\sin^2 2\theta_{23}$	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$
A_4 Tetrahedral Models:					
Ma1 [35]	NH	1.0	0.31	0	0.50
	IH	1.0	0.33 - 0.34	0	0.50
ABGMP [36]	NH	1.0	0.27 - 0.30	0.0007 - 0.0037	0.51 - 0.52
AG1 [37]	NH	1.0	0.31	0.0026 - 0.034	0.51 - 0.56
HT [38]	NH	1.0	0.29 - 0.33	< 0.0022	
AG2 [39]	IH	1.0	0.27 - 0.34	< 0.0012	0.52 - 0.53
L [40]	NH	1.0	0.29 - 0.38	0.0025	
Ma2 [41]	NH	1.0	0.32	0	0.50
S_4 Models:					
MPR [42]	Q-deg	0.99	0.25 - 0.37	0.008 - 0.01	0.44
HLM [43]	NH	1.0	0.30	0.0044	0.50
	NH	1.0	0.31	0.0034	0.50
Z [44]	NH	0.96 - 1.0	0.311	< 0.030	0.41 - 0.50
$SO(3)$ Models:					
M [45]	NH	1.0	0.31	0.00005	
W [46]	NH			0.0027 - 0.036	
T' Models:					
FM [47]	NH	0.93 - 0.95		0.024 - 0.036	

Table 2: Mixing Angles for Models with Sequential Right-Handed Neutrino Dominance.

Reference	Flavor Sym.	Hierarchy	$\sin^2 2\theta_{23}$	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$
D [48]	Z_3	NH			0.008 - 0.14	
K [49]	SO(3)	NH	0.99 - 1.0	0.28 - 0.39	0.0027	
H [50]		NH	1.0	0.30	0.0033	0.52
EH [51]	U(1)	NH	0.99	0.31	0.0009	0.54

5. Survey of Mixing Angle Predictions

The author has updated a previous survey[1] made in collaboration with Mu-Chun Chen in 2006 of models in the literature which satisfied the then current experimental bounds on the mixing angles and gave reasonably restrictive predictions for the reactor neutrino angle. The cutoff date for the present update is January 2009.

Many models in the literature lack firm predictions for any of the mixing angles. For our analyzis no requirement is made that the solar and atmospheric mixing angles or the mass differences be predicted, but if so, they must also satisfy the bounds given in Eq. (1). The 86 models which meet our criteria are listed in Tables 1 - 4.

Table 3: Mixing Angles for $SO(10)$ Models with Symmetric/Antisymmetric Contributions.

Reference	Flavor Sym.	Hier.	$\sin^2 2\theta_{23}$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{13}$	$\sin^2 \theta_{23}$
BRT [52]	U(2) x U(1)^n	NH	0.99	0.26	0.0024	0.55
BW [53]		NH			O(0.01)	
SP [54]		NH	0.99	0.30	0.0002	0.50
Ra [55]	SU(2) x U(1)	NH			O(0.01)	
BO [56]	U(1)$_A$	NH	≥ 0.95	0.19 - 0.38	0.0014	
O [57]		NH	0.94	0.31	0.0007	
KR [58]	SU(3) x R x U(1) x Z_2	NH	0.93	0.30	0.058	0.63
Ro [59]		NH IH			0.0056 0.036	
GMN2 [60]		NH	≤ 0.91	≥ 0.34	0.026	
YW [61]		NH	0.96	0.29	0.04	
CM1 [62]	SU(2) x Z_2 x Z_2 x Z_2	NH	1.0	0.26	0.014	0.51
BeM [63]		NH	0.93	0.29	0.012	0.53
BaM [64]		NH	0.98	0.31	0.013	
DR [65]	D_3 x U(1) x Z_2 x Z_2	NH	0.99	0.29	0.0024	0.55
VR [66]	SU(3)	NH	≥ 0.99	0.29 - 0.38	0.024	0.44 - 0.56
DMM [67]		NH			0.0036 - 0.012	
ShT [68]	R sym.	NH	0.99	0.31	0.0001 - 0.04	0.44
BN [69]	SU(3)	NH	1.0	0.26-0.28	0.0009 - 0.016	0.5 - 0.51
BMSV [70]		IH			≥ 0.01	
DHR [71]	D_3	NH	1.0	0.29	0.0025 - 0.0037	0.53 - 0.54
KM [72]	SO(3) 5D	NH		0.30 - 0.37	0.0012	
CY [73]	S_4	NH	1.0	0.28	0.0029	0.53
GK1 [74]	Z_2	NH		0.031	0.01	
FMN [75]		NH	1.0	0.32	0.0002	0.53
GK2 [76]	A_4	NH IH	≥ 0.96	0.25 - 0.5 0.28 - 0.5	0.0002 0.0025	0.4 - 0.7 0.3 - 0.7
Mo [77]		NH	0.97	0.35	0.017	0.42
P [78]	S_4	NH	1.0	0.26 - 0.38	0.0027 - 0.0032	0.52 - 0.54
BR [79]		NH			0.0027 - 0.024	

Table 4: Mixing Angles for $SO(10)$ Models (or otherwise indicated) with Lopsided Mass Matrices.

Reference	Flavor Sym.	Hier.	$\sin^2 2\theta_{23}$	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$
Mae [80]	U(1)	NH			0.048	
AB [81]	U(1) x $(Z_2)^2$	NH	0.99	0.33	0.0002	0.54
BB [82]		NH	0.97	0.29	0.0016 - 0.0025	0.58
A [83]	U(1) x $(Z_2)^2$	NH	0.99	0.28	0.0022	0.55
JLM [84]		NH	1.0	0.29	0.019	0.49
* CM2 [85]	T′ x $(Z_2)^2$	NH	1.0	0.30	0.0030	0.50
† StT [86]	SU(3) x Z_2	IH	1.00	0.31	0.012	0.47
FHLR [87]		NH			0.05	
		IH			$\lesssim 0.01$	
HSS [88]	$\Sigma(81)$	NH	1.0	0.27	0.0004	0.53

* $SU(5)$ based model
† E_6 based model

Histograms are plotted against $\sin^2\theta_{13}$, where all models are given the same area, even if they extend across several basic intervals. The results are shown in Figs. 1 and 2 for the lepton flavor models and grand unified models, respectively. Two thirds of both types of models predict $0.001 \lesssim \sin^2\theta_{13} \lesssim 0.05$, while the lepton flavor models have a much longer tail extending to very small reactor neutrino angles. The planned experiments involving Double CHOOZ and Daya Bay reactors[89] will reach down to $\sin^2 2\theta_{13} \lesssim 0.01$, so roughly two-thirds of the models will be eliminated if no $\bar{\nu}_e$ depletion is observed. Both the T2K Collaboration at JPARC and the NOνA Collaboration at Fermilab are also expected to probe a similar reach with their ν_μ neutrino beams[90].

Even if $\bar{\nu}_e$ depletion is observed with some accuracy, it is apparent from the two histograms that the order of 10 - 20 models may survive which must still be differentiated. One suggestion is to make scatterplots of $\sin^2\theta_{13}$ *vs.* $\sin^2\theta_{12}$ and $\sin^2\theta_{12}$ *vs.* $\sin^2\theta_{23}$. We have attempted to do this in Figs. 3, 4, and 5 for both the lepton flavor models and grand unified models. Note that even fewer of the 86 models considered make predictions for the solar and atmospheric neutrino mixing angles. In addition, we emphasize that only the central value predictions have been plotted, while some of the models have rather large theoretical error bars associated with them.

Still one can make some interesting conclusions. In particular, most of the models considered favor central values of $\sin^2\theta_{12}$ lying below 0.333, the value for exact tri-bimaximal mixing. This is in agreement with the present value extracted in Eq. (1). But perhaps even more surprising is that central values for $\sin^2\theta_{23} \geq 0.5$ are preferred, while the best extracted value of 0.466 from Eq. (1) lies below 0.5.

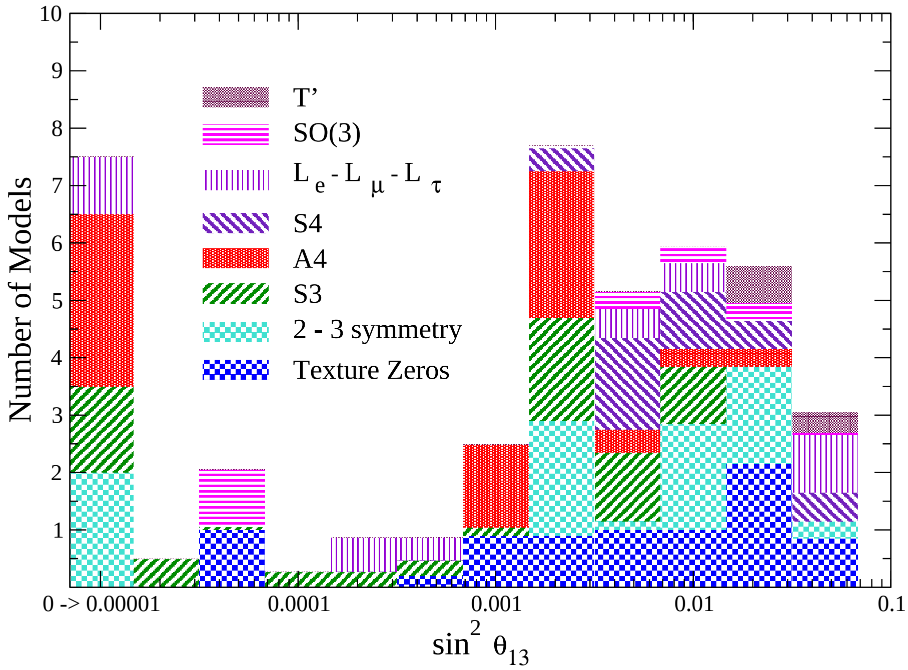

Figure 1: Predictions of $\sin^2\theta_{13}$ for the lepton flavor models considered.

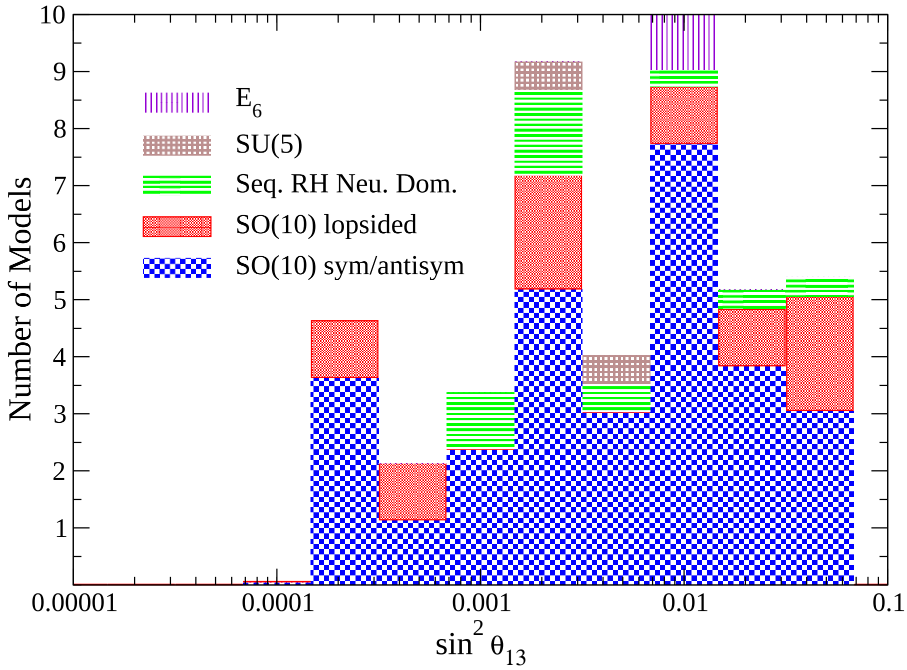

Figure 2: Predictions of $\sin^2\theta_{13}$ for the SO(10) models considered.

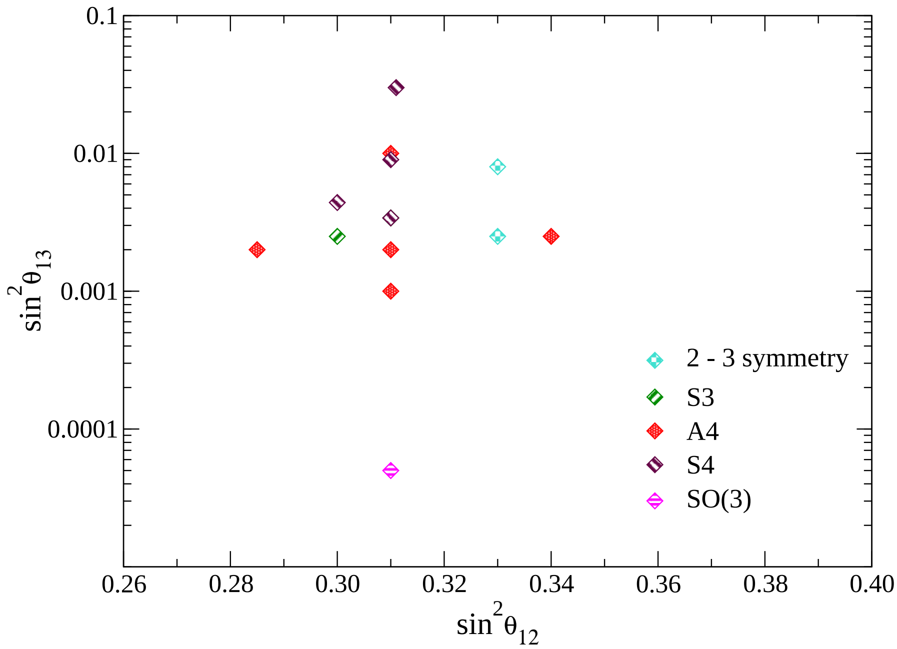

Figure 3: Predictions of the $\sin^2\theta_{23}$ *vs.* $\sin^2\theta_{12}$ distribution of central values for the discrete flavor symmetry models considered.

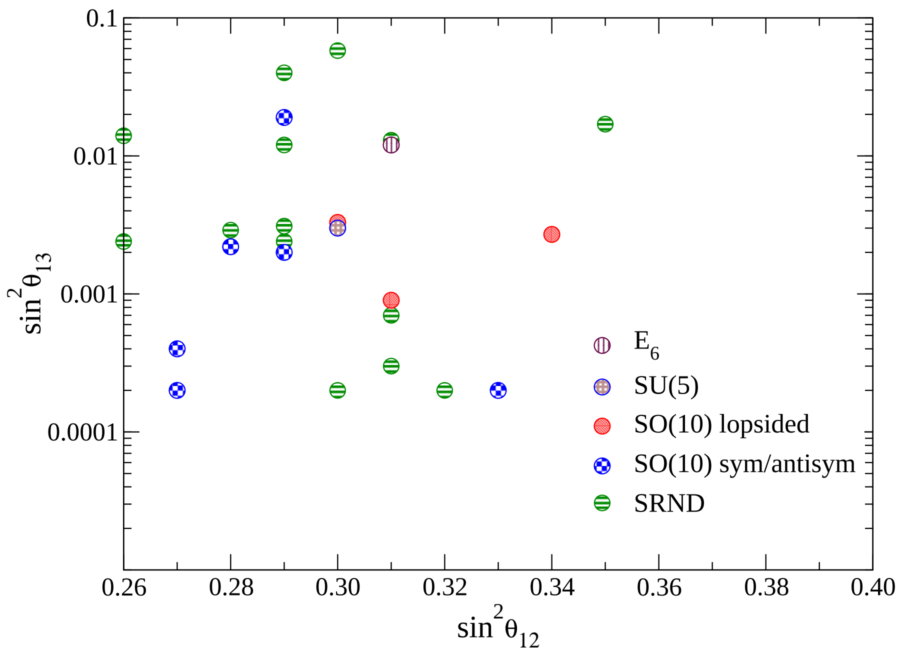

Figure 4: Predictions of the $\sin^2\theta_{23}$ *vs.* $\sin^2\theta_{12}$ distribution of central values for the grand unified models considered.

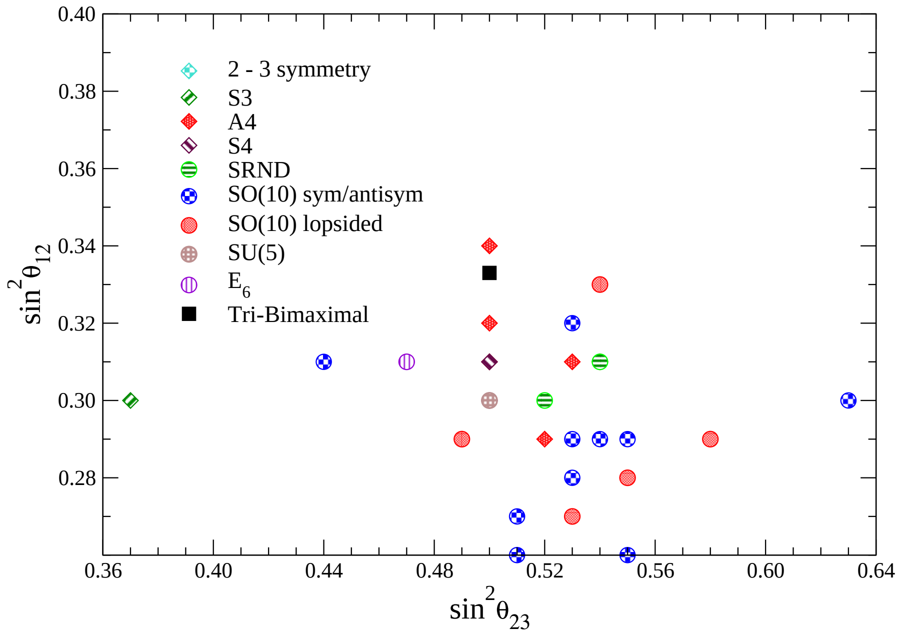

Figure 5: Predictions of the $\sin^2\theta_{12}$ *vs.* $\sin^2\theta_{23}$ distribution of central values for both types of models considered.

6. Other Tests

6.1. Nature of Tri-bimaximal Mixing

As pointed out earlier, many of the GUT models were based on continuous and/or discrete flavor symmetries with no aim in mind to reproduce tri-bimaximal mixing at leading order. This raises the issue whether tribimaximal mixing is a hidden symmetry which is softly broken or just an accidental symmetry of nature.

In order to pursue this issue, the author in collaboration with Werner Rodejohann adopted a model-independent approach[91)]. In the lepton flavor basis, deviations from tri-bimaximal mixing were considered by perturbing each element of the neutrino mass matrix by up to 20%:

$$m_\nu = \begin{pmatrix} A(1+\epsilon_1) & B(1+\epsilon_2) & B(1+\epsilon_3) \\ \cdot & \frac{1}{2}(A+B+D)(1+\epsilon_4) & \frac{1}{2}(A+B-D)(1+\epsilon_5) \\ \cdot & \cdot & \frac{1}{2}(A+B+D)(1+\epsilon_6) \end{pmatrix} \tag{12}$$

Recall that for TBM mixing, $A = \frac{1}{3}(2m_1 + m_2 e^{-2i\alpha})$, $\quad B = \frac{1}{3}(m_2 e^{-2i\alpha} - m_1)$, $\quad D = m_3 e^{-2i\beta}$.

Scatterplots are then constructed with points chosen according to the following prescription: Start with the central best values for the mass differences in Eq. (1),

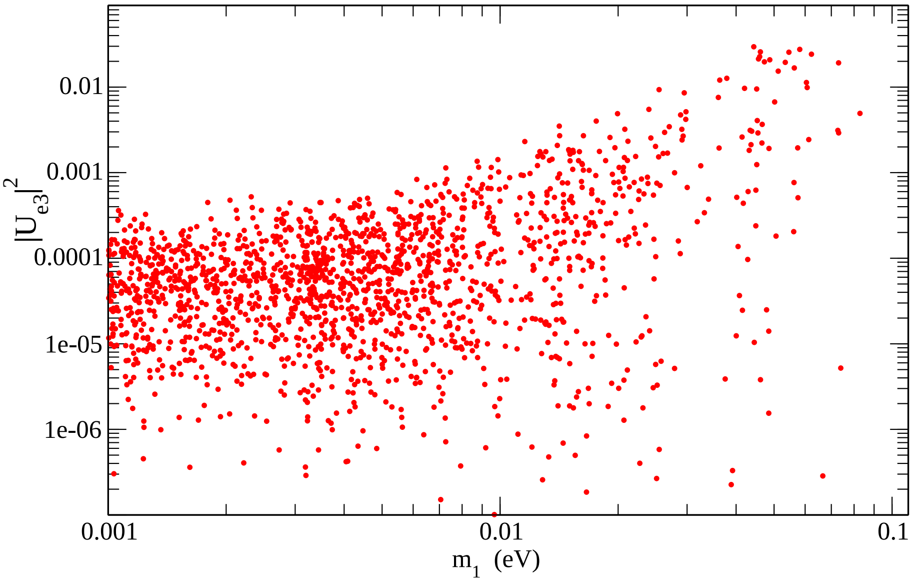

Figure 6: Scatterplot for $\sin^2\theta_{13}$ *vs.* m_1 distribution for normal ordering of perturbed tri-maximal mixing.

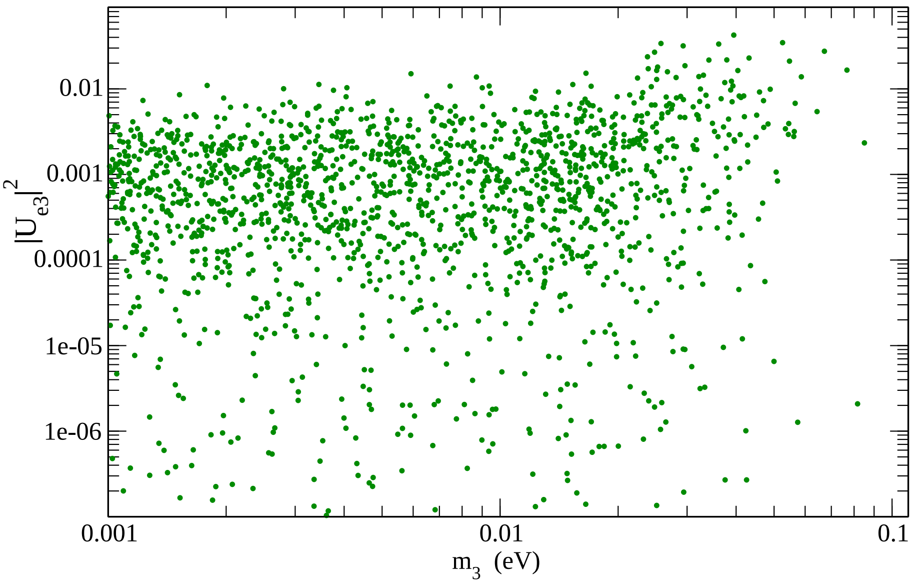

Figure 7: Scatterplot for $\sin^2\theta_{13}$ *vs.* m_1 distribution for inverted ordering of perturbed tri-maximal mixing.

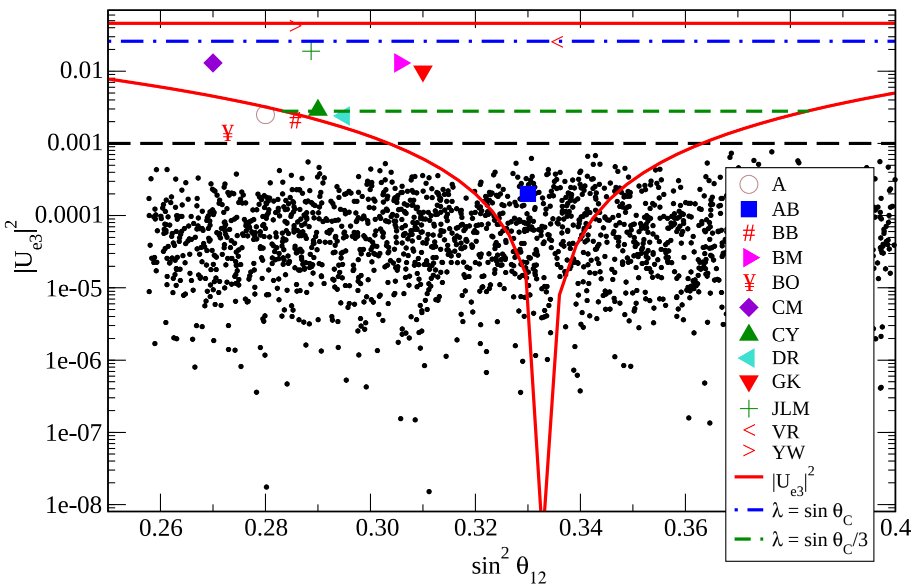

Figure 8: Scatterplot for $\sin^2\theta_{13}$ $vs.$ $sin^2\theta_{12}$ for perturbed TBM mixing and GUT model predictions.

hold m_3 fixed for normal hierarchy or m_2 fixed for inverted hierarchy, and let the other masses vary by up to 20%; vary the Majorana phases in their full ranges; and vary each ϵ_i within $|\epsilon_i| \leq 0.2$ for its full phase range. For each choice of parameters the resulting mass matrix is diagonalized and, if the outcome is within the current 3σ ranges quoted in Eq. (1), the point is kept.

The resulting scatterplots of $|U_{e3}|^2$ $vs.$ m_1 for normal ordering and $vs.$ m_3 for inverted ordering are shown in Figs. 6 and 7, respectively. From Fig. 6 one sees that $|U_{e3}|^2$ remains below 0.001 for all $m_1 < 4.5$ meV, corresponding to a normal hierarchy, and only increases above that value once larger values of m_1 appear, corresponding to normal ordering until quasi-degenerate neutrino masses occur. In Fig. 7 for the inverted hierarchy and ordering, the corresponding bound is noticeably higher at 0.01.

The scatterplot in Fig. 8 of $|U_{e3}|^2$ $vs.$ $\sin^2\theta_{12}$ applies for the normal hierarchy case with a fixed value of $m_3 = 0.050$ eV. Again it is apparent that all points lie below 0.001. This suggests that for a normal hierarchy of neutrino masses, tri-bimaximal mixing is accidental, if $\sin^2\theta_{13}$ is found experimentally to be larger than the bounded deviation from zero of 0.001. No such statement can be made for an inverted hierarchy, for the restricted bound is much weaker for deviations from TBM mixing and can essentially extend up to nearly the present experimental limit. Also note from Fig. 8 that no restrictions are placed on deviations of $\sin^2\theta_{12}$ from the TBM value of 0.333.

For comparison, we also show the results for twelve GUT models. Note that for all but one, $\sin^2\theta_{13}$ is projected to lie above the softly-broken TBM mixing bound of 0.001.

However, if the charged lepton flavor matrix is rotated by one-third the Cabibbo angle (or by the Cabibbo angle itself) from its original diagonal form, while the neutrino matrix keeps the TBM form, one finds a larger deviation of $\sin^2\theta_{13} = 0.0029$ (0.025). These limits are depicted by the dashed and broken lines, respectively, in Fig. 8. For an arbitrary 12 rotation of the charged lepton mass matrix from the diagonal form, the acceptable points lie between the solid line boundaries.

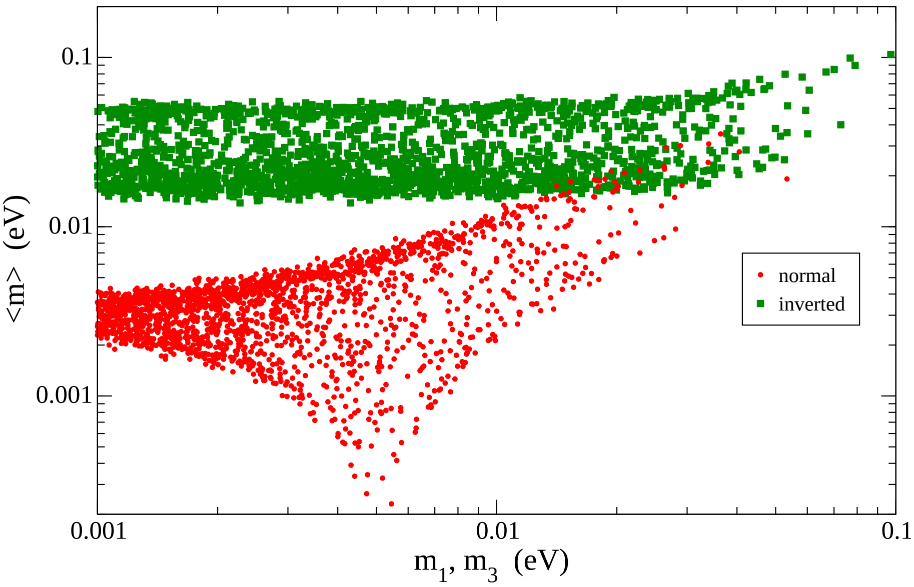

Figure 9: Effective mass plot for neutrino-less double beta decay in the case of perturbed tri-bimaximal mixing.

6.2. Neutrino-less Double Beta Decay

Neutrino-less double beta decay provides an opportunity to test the Majorana *vs.* Dirac nature of the light neutrinos and whether the mass ordering is normal or inverted in the former case. The square of the effective mass entering the decay rate is given by

$$\begin{aligned} \langle m_{\beta\beta}\rangle &= \left|\Sigma_i m_i U_{ei}^2 \Phi_{ii}^2\right| \\ &= m_1 U_{ei}^2 + m_2 U_{e2}^2 e^{i2\alpha} + m_3 U_{e3}^2 e^{i2\beta} \\ &\simeq m_1 \cos^2\theta_{12} + m_2 \sin^2\theta_{12} e^{i2\alpha}, \end{aligned} \tag{13}$$

where it is apparent the Majorana phases play an important role. Since $m_1 \sim m_2 \gg m_3$ for the inverted hierarchy case, the $(Z, A) \to (Z+2, A) + 2e^-$ process should occur with a shorter lifetime than for the normal hierarchy case. We show in Fig. 9 the effective mass plot as a function of the lightest neutrino mass, m_1 (m_3), in the normal (inverted) ordering case. The plots were obtained for tri-bimaximal mixing perturbed as described in the previous subsection. There is a rather clear separation of the normal and inverted ordering distributions.

6.3. Charged Lepton Flavor Violation

Charged lepton flavor violation provides one more way to differentiate neutrino mixing models, if the Dirac and right-handed Majorana neutrino mass matrices are specified[92)]. Of special interest are the limits on the branching ratios for $\mu \rightarrow e + \gamma$ and $\mu - e$ conversion, for example. The former decay branching ratio is presently under test by the MEG experiment[93)] which plans to lower the present bound of 1.2×10^{-11} to $3 - 5 \times 10^{-13}$. No $\mu - e$ conversion experiment is presently underway, although plans for one exist at both J-PARC and Fermilab.

7. Summary

We have made a survey of neutrino mixing models based on some horizontal lepton flavor symmetry and those based on GUT models having a vertical family symmetry and a flavor symmetry. We have tried to differentiate the models on the basis of their neutrino mass hierarchy, mixing angles, and neutrino-less double beta predictions. Most of the models allow either mass hierarchy with the exceptions being just normal for the type I seesaw models and only inverted for the conserved $L_e - L_\mu - L_\tau$ models.

For both types of models our study indicates that the upcoming Double CHOOZ and Daya Bay reactor experiments will be able to eliminate roughly two-thirds of the models surveyed, if their planned sensitivity reaches $\sin^2 2\theta_{13} \simeq 0.001$ and no depletion of the $\bar{\nu}_e$ flux is observed. However, no smoking gun apparently exists to rule out many types of models based on accurate data for $\sin^2 \theta_{13}$ alone, should evidence for a depletion be found. Of the order of 10 - 20 models have similar values for this mixing angle in the 0.001 - 0.05 interval. These results for the $\sin^2 \theta_{13}$ distributions involve more models but are somewhat similar to those obtained in an earlier survey published in 2006 in collaboration with Mu-Chun Chen[1)]. Only the lepton flavor models appear to lead to extremely small values of $\sin^2 \theta_{13} \lesssim 10^{-4}$.

Most models prefer $\sin^2 \theta_{12} \lesssim 0.31$ rather than 0.333 for tri-bimaximal mixing in agreement with the present best value of 0.312. On the other hand, most models prefer $\sin^2 \theta_{23} \geq 0.50$ compared with a best fit value of 0.466.

Effective mass plots for perturbed tri-bimaximal mixing show a clear separation of the normal and inverted ordering distributions, so accurate neutrino-less double beta decay experiments should be decisive.

It is clear that very accurate determination of the three mixing angles and eventually the three CP-violating phases will be required to pin down the most viable models.

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